J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

MATRICES + TOPIC : CORRELATION

SYJC TEST - 01 - SET 2

DURATION - 1 $^1/_2$ HR

MARKS - 40

SOLUTION SET

SECTION - I

Q-1

01. $X + Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix}$ and $X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

Find the matrices Y

SOLUTION

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$-\quad + \quad -$

$$2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$Y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

02. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & a & 2 \\ 5 & 7 & 3 \end{bmatrix}$ is a singular matrix
Find a

SOLUTION

Since A is singular matrix

$|A| = 0$

$1(3a - 14) - 2(6 - 10) + 3(14 - 5a) = 0$

$3a - 14 - 2(-4) + 42 - 15a = 0$

$3a - 14 + 8 + 42 - 15a = 0$

$36 - 12a = 0$

$\therefore a = 3$

03. if $A = \begin{bmatrix} 7 & 1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$;

Verify : $|AB| = |A|.|B|$

SOLUTION

LHS

AB

$$= \begin{bmatrix} 7 & 1 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 + 3 & 14 - 1 \\ 2 + 15 & 4 - 5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 13 \\ 17 & -1 \end{bmatrix}$$

$|AB| = 10(-1) - 17(13) = -10 - 221$

$= -231$

RHS

$= |A| . |B|$

$$= \begin{vmatrix} 7 & 1 \\ 2 & 5 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$= (35 - 2) . (-1 - 6)$

$= 33 (-7)$

$= -231$ LHS = RHS

04. find the ADJOINT of the matrix

$$A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$$

COFACTOR'S

$$A_{11} = (-1)^{1+1}\,5 = 5$$

$$A_{12} = (-1)^{1+2}\,3 = -3$$

$$A_{21} = (-1)^{2+1}\,(-3) = 3$$

$$A_{22} = (-1)^{2+2}\,2 = 2$$

COFACTOR MATRIX OF A

$$= \begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}$$

ADJ A

= TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$$

Q2. Attempt any TWO of the following

(3 marks each)

01.

$$[-1 \;\; 1 \;\; 4]\;\; 2\begin{bmatrix} 5 & 5 \\ 6 & 6 \\ -1 & 2 \end{bmatrix} + 3\begin{bmatrix} 3 & 4 \\ 4 & 1 \\ 1 & -1 \end{bmatrix} = [x \;\; y]$$

$$[-1 \;\; 1 \;\; 4]\begin{bmatrix} 10 & 10 \\ 12 & 12 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 12 \\ 12 & 3 \\ 3 & -3 \end{bmatrix} = [x \;\; y]$$

$$[-1 \;\; 1 \;\; 4]\begin{bmatrix} 10+9 & 10+12 \\ 12+12 & 12+3 \\ -2+3 & 4-3 \end{bmatrix} = [x \;\; y]$$

$$[-1 \;\; 1 \;\; 4]\begin{bmatrix} 19 & 22 \\ 24 & 15 \\ 1 & 1 \end{bmatrix} = [x \;\; y]$$

$$[-19 + 24 + 4 \quad -22 + 15 + 4] = [x \;\; y]$$

$$[9 \;\; -3] = [x \;\; y]$$

By equality of two matrices

$$x = 9 \;\;\&\;\; y = -3$$

02.

$$x + 2y + z = 8$$

$$2x + 3y - z = 11$$

$$3x - y - 2z = 5$$

$$AX = B$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 5 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 5 \end{bmatrix}$$

$R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & -7 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -19 \end{bmatrix}$$

$R_2 \times (-1)$ & $R_3 \times (-1)$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 7 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 19 \end{bmatrix}$$

$R_3 - 7R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -16 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y + z \\ y + 3z \\ -16z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ -16 \end{bmatrix}$$

BY EQUALITY OF TWO MATRICES

$-16z = -16 \quad \therefore\; z = 1$

$y + 3z = 5$

subs $z = 1 \quad \therefore\; y = 2$

$x + 2y + z = 8$

subs $y = 2$ & $z = 1$ $\therefore\; x = 3$

SS : { 3 , 2 , 1 }

03. $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ Show that : $A^2 - 5A + 7I = 0$

Hence find : A^{-1}

Q-2

STEP 1

$$LHS = A^2 - 5A + 7I$$

$$= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} - 5 \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8-15+7 & 5-5+0 \\ -5+5+7 & 3-10+7 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = RHS$$

STEP 2 SOLVING FOR A^{-1}

$|A| = 6 + 1 = 7 \neq 0$ Hence A^{-1} exists

$$A^2 - 5A + 7I = 0$$

$$A^2 A^{-1} - 5A A^{-1} + 7 IA^{-1} = 0$$

$$AA A^{-1} - 5A A^{-1} + 7 IA^{-1} = 0$$

$$AI - 5I + 7A^{-1} = 0$$

$$A - 5I + 7A^{-1} = 0$$

$$7A^{-1} = 5I - A$$

$$7A^{-1} = 5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$7A^{-1} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$7A^{-1} = \begin{pmatrix} 5-3 & 0-1 \\ 0+1 & 5-2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{7} \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$$

Q - 3

Q3. Attempt any TWO of the following (4 marks each)

01. $A = \begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{pmatrix}$

$$\begin{aligned} |A| &= 1(-0-3) - 2(0+1) - 2(0-2) \\ &= 1(-3) - 2(1) - 2(-2) \\ &= -3 - 2 + 4 \\ &= -1 \\ &\neq 0 \quad \text{Hence } A^{-1} \text{ exists} \end{aligned}$$

$AA^{-1} = I$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R_3 + R_1$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ 0 & 5 & -2 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

R_{23}

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_2 + 2R_3$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_1 - 2R_2$

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} -1 & -4 & -2 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_3 + 2R_2$

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} -1 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$R_1 + 2R_3$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$$I.A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

Q-3

02. $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$

Verify : A.(adj A) = (adj A).A = |A|.I

COFACTOR'S

$A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -2 \\ 0 & 2 \end{vmatrix} = 1(0 - 0) = 0$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -1(9 + 2) = -11$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 1(0 - 0) = 0$

$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -1(-3 - 0) = 3$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1(3 - 2) = 1$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -1(0 + 1) = -1$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 1(2 - 0) = 2$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -1(-2 - 6) = 8$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 1(0 + 3) = 3$

COFACTOR MATRIX OF A

$$\begin{pmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{pmatrix}$$

ADJ A = TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

|A|

$= 1(0 + 0) + 1(9 + 2) + 2(0 - 0)$

$= 11$

LHS

= A.(adj A)

$$= \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 11 + 0 & 3 - 1 - 2 & 2 - 8 + 6 \\ 0 - 0 - 0 & 9 + 0 + 2 & 6 + 0 - 6 \\ 0 - 0 + 0 & 3 + 0 - 3 & 2 + 0 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

RHS

= |A|.I

$$= 11 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

HENCE A.(adj A) = (adj A).A = |A|.I

03. $A = \begin{pmatrix} 1 & 2 \\ -1 & -2 \end{pmatrix}$ $B = \begin{pmatrix} 2 & a \\ -1 & b \end{pmatrix}$ **Q - 3**

such that :$(A + B)^2 = A^2 + B^2$,find a & b

SOLUTION

$(A + B)^2 = A^2 + B^2$

$(A + B)(A + B) = A^2 + B^2$

$A^2 + AB + BA + B^2 = A^2 + B^2$

$AB + BA = 0$

$$\begin{pmatrix} 1 & 2 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 2 & a \\ -1 & b \end{pmatrix} + \begin{pmatrix} 2 & a \\ -1 & b \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 - 2 & a + 2b \\ -2 + 2 & -a - 2b \end{pmatrix} + \begin{pmatrix} 2 - a & 4 - 2a \\ -1 - b & -2 - 2b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 + 2 - a & a + 2b + 4 - 2a \\ 0 - 1 - b & -a - 2b - 2 - 2b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 - a & -a + 2b + 4 \\ -1 - b & -a - 4b - 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

By equality of two matrices

$2 - a = 0 \quad \therefore \quad a = 2$

$-1 - b = 0 \quad \therefore \quad b = -1$

SECTION - II

Q-4

Q4.1. $n = 100$; $\overline{x} = 62$; $\overline{y} = 53$;

$\sigma x = 10$; $\sigma y = 12$

$\Sigma(x - \overline{x})(y - \overline{y}) = 8000$

SOLUTION

$$r = \frac{cov(x,y)}{\sigma x \, . \, \sigma y}$$

$$= \frac{\frac{\Sigma(x - \overline{x})(y - \overline{y})}{n}}{\sigma x \, . \, \sigma y}$$

$$= \frac{\frac{8000}{100}}{10.12}$$

$$= \frac{80}{10.12}$$

$$= \frac{2}{3}$$

02. Ranking of 8 trainees at the beginning (X) and at the end (Y) of a certain course are given below Find the rank correlation

SOLUTION

	x	y	d = |x − y|	d^2
A	1	2	1	1
B	2	4	2	4
C	4	3	1	1
D	5	7	2	4
E	6	8	2	4
F	8	1	7	49
G	3	5	2	4
H	7	6	1	1
				$\Sigma d^2 = 68$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(68)}{7(49 - 1)}$$

$$= 1 - \frac{6(68)}{7(48)}$$

$$= 1 - \frac{17}{14}$$

$$= \frac{-3}{14} = -0.21$$

03. coefficient of correlation between variables X and Y is 0.3 and their covariance is 12 . The variance of X is 9 . Find standard deviation of Y

SOLUTION

$r = 0.3$, $cov(x,y) = 12$, $\sigma x^2 = 9$,

$$r = \frac{cov(x,y)}{\sigma x \, . \, \sigma y}$$

$$0.3 = \frac{12}{3 \times \sigma y}$$

$$\sigma y = \frac{12}{3 \times 0.3}$$

$$= \frac{120}{3 \times 3}$$

$$= \frac{40}{3} = 13.33$$

04. The coefficient of rank correlation for a certain group of data is 0.5 . If $\Sigma d^2 = 42$, assuming no ranks are repeated ; find the no. of pairs of observation

SOLUTION

$R = 0.5$; $\Sigma d^2 = 42$

$$\mathbf{R} = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.5 = 1 - \frac{6(42)}{n(n^2 - 1)}$$

$$\frac{6(42)}{n(n^2 - 1)} = 1 - 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = \frac{1}{2}$$

$$n(n^2 - 1) = 6 \times 42 \times 2$$

$$n(n^2 - 1) = 3 \times 2 \times 3 \times 2 \times 7 \times 2$$

$$(n - 1).n.(n + 1) = 7 \times 8 \times 9$$

On comparing , $n = 8$

Q-5

1. find number of pair of observations from the following data

$r = 0.5$; $\sum xy = 120$; $SDy = 8$

$\sum x^2 = 90$; where x and y are deviations from their respective means

SOLUTION

$r = 0.5$; $\sum(x - \overline{x})(y - \overline{y}) = 120$;

$\sigma y = 8$; $\sum(x - \overline{x})^2 = 90$

$\sigma y = 3$

$$\sqrt{\frac{\Sigma(y - \overline{y})^2}{n}} = 8$$

$$\sqrt{\Sigma(y - \overline{y})^2} = 8\sqrt{n}$$

NOW

$$r = \frac{\Sigma (x - \overline{x}).(y - \overline{y})}{\sqrt{\Sigma(x - \overline{x})^2}\sqrt{\Sigma(y - \overline{y})^2}.}$$

$$0.5 = \frac{120}{\sqrt{90} \;.\; 8\sqrt{n}}$$

$$\frac{1}{2} = \frac{120}{\sqrt{90} \;.\; 8\sqrt{n}}$$

$$\sqrt{n} = \frac{120 \times 2}{\sqrt{90} \;.\; 8}$$

$$\sqrt{n} = \frac{30}{\sqrt{90}}$$

Squaring ;

$$n = \frac{900}{90} = 10$$

02

$N = 25$; $\sum x = 75$; $\sum y = 100$; $\sum x^2 = 250$

$\sum y^2 = 500$; $\sum xy = 325$

SOLUTION

$$\mathbf{r = \frac{n\Sigma xy - \Sigma x.\Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\;\sqrt{n\Sigma y^2 - (\Sigma y)^2}}}$$

$$r = \frac{25(325) - 75(100)}{\sqrt{25(250) - 75^2}\;\sqrt{25(500) - 100^2}}$$

$$r = \frac{8125 - 7500}{\sqrt{6250 - 5625}\;\sqrt{12500 - 10000}}$$

$$r = \frac{625}{\sqrt{625}\;\sqrt{2500}}$$

$$r = \frac{625}{25 \times 50}$$

$$r = \frac{1}{2} = 0.5$$

03. the coefficient of rank correlation of marks obtained by 10 students in English and Economics was found to be 0.5. It was later found that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 3 instead of 7 . Find the correct coefficient of rank correlation

Q-5

SOLUTION

N = 10 , R = 0.5

Incorrect d = 3 while correct d = 7

STEP – 1

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.5 = 1 - \frac{6\Sigma d^2}{10(100 - 1)}$$

$$0.5 = 1 - \frac{6\Sigma d^2}{10(99)}$$

$$0.5 = 1 - \frac{\Sigma d^2}{165}$$

$$\frac{\Sigma d^2}{165} = 1 - 0.5$$

$$\frac{\Sigma d^2}{165} = 0.5$$

$$\Sigma d^2 = 82.5$$

STEP 2

$$\Sigma d^2 = 82.5$$

$$-3^2 \qquad -9$$

$$+40$$

$$+7^2 \qquad +49$$

$$\Sigma d^2_{correct} = 122.5$$

STEP 3

$$R_{correct} = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(122.5)}{10(100 - 1)}$$

$$= 1 - \frac{6(122.5)}{10(99)}$$

$$= 1 - \frac{245}{330}$$

$$= 1 - \frac{49}{66}$$

$$= \frac{17}{66}$$

$$= 0.2575$$

Q-6

01.

x : 6 2 10 4 8

y : 9 11 5 8 7 . Find Karl Pearson's Correlation coeff.

x	y	$x-\overline{x}$	$y-\overline{y}$	$(x-\overline{x})^2$	$(y-\overline{y})^2$	$(x-\overline{x})(y-\overline{y})$
6	9	0	1	0	1	0
2	11	–4	3	16	9	–12
10	5	4	–3	16	9	–12
4	8	–2	0	4	0	0
8	7	2	–1	4	1	–2
30 Σx	40 Σy	0 $\Sigma(x-\overline{x})$	0 $\Sigma(y-\overline{y})$	40 $\Sigma(x-\overline{x})^2$	20 $\Sigma(y-\overline{y})^2$	–26 $\Sigma(x-\overline{x})(y-\overline{y})$
$\overline{x} = 6$ $\overline{y} = 8$						

$$r = \frac{\Sigma (x-\overline{x}).(y-\overline{y})}{\sqrt{\Sigma(x-\overline{x})^2}\ \sqrt{\Sigma(y-\overline{y})^2}}.$$

$$r = \frac{-26}{\sqrt{40} \times \sqrt{20}}$$

$$r = \frac{-26}{\sqrt{40 \times 20}} \qquad \text{(Since log a , a > 0)}$$

let

$$r' = \frac{26}{\sqrt{40 \times 20}}$$

taking log on both sides

$$\log r' = \log 26 - \frac{1}{2}\left(\log 40 + \log 20\right)$$

$$\log r' = 1.4150 - \frac{1}{2}[\ 1.6021 + 1.3010\ ]$$

$$\log r' = 1.4150 - \frac{1}{2}(2.9031)$$

$$\log r' = 1.4150 - 1.4516$$

$$\log r' = \overline{1}.9634$$

$$r' = AL(\overline{1}.9634)$$

$$r' = 0.9191$$

$$r = -0.9191$$

02. Given X : 1 2 3 …………. n Show that : $Cov(x,y) = \frac{n^2 - 1}{12}$

Y : 1 2 3 ………… n

SOLUTION

$$Cov(x,y) = \frac{\Sigma xy}{n} - \bar{x} . \bar{y}$$

STEP 1 : Find means $\bar{x}$ & $\bar{y}$

$$\Sigma x = 1 + 2 + 3 + …….. n = \frac{n(n+1)}{2}$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$\Sigma y = 1 + 2 + 3 + …….. n = \frac{n(n+1)}{2}$$

$$\bar{y} = \frac{\Sigma x}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

STEP 2 : $\frac{\Sigma xy}{n}$

$$= \frac{1.1 + 2.2 + 3.3 + ……… + n.n}{n}$$

$$= \frac{1^2 + 2^2 + 3^2 + ………. + n^2}{n}$$

$$= \frac{n(n+1)(2n+1)}{6n}$$

$$= \frac{(n+1)(2n+1)}{6}$$

STEP 3 : $cov(x,y) = \frac{\Sigma xy}{n} - \bar{x} . \bar{y}$

$$= \frac{(n+1)(2n+1)}{6} - \frac{n+1}{2} . \frac{n+1}{2}$$

$$= \frac{n+1}{2} \left(\frac{2n+1}{3} - \frac{n+1}{2} \right)$$

$$= \frac{n+1}{2} \left(\frac{4n + 2 - 3n - 3}{6} \right)$$

$$= \frac{n+1}{2} \frac{n-1}{6}$$

$$= \frac{n^2 - 1}{12}$$

03.

SOLUTION

X : Marks in test 1

Y : Marks in test 2

X Test1	Y Test 2	x	y	d $=\|x-y\|$	d^2
52	# 65	3	4.5	1.5	2.25
*34	59	6.5	6	0.5	0.25
47	# 65	4	4.5	0.5	0.25
65	68	1	3	2	4
43	82	5	1	4	16
*34	80	6.5	2	4.5	20.25
54	57	2	7	5	25
					$\Sigma d^2 = 68$

CORRECTION FACTORS

Using $\frac{m(m^2-1)}{12}$

$$T_x = \frac{2(2^2-1)}{12} = \frac{2(3)}{12} = 0.5$$

$$T_y = \frac{2(2^2-1)}{12} = \frac{2(3)}{12} = 0.5$$

$$\mathbf{\Sigma d^2 \text{ corrected}} = \Sigma d^2 + Tx + Ty$$

$$= 68 + 0.5 + 0.5$$

$$= 69$$

$$\mathbf{R} = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6(69)}{7(49-1)}$$

$$= 1 - \frac{6(69)}{7(48)}$$

$$= 1 - \frac{69}{56}$$

$$= \frac{-13}{56} = -0.23$$